Calhoun: The NPS Institutional Archive
DSpace Repository

Theses and Dissertations 1. Thesis and Dissertation Collection, all items

1985

Application of life distributions to estimate equipment losses in combat.

Song, Hee S.

http://hdl.handle.net/10945/21166

Downloaded from NPS Archive: Calhoun

Calhoun is the Naval Postgraduate School's public access digital repository for research materials and institutional publications created by the NPS community. Calhoun is named for Professor of Mathematics Guy K. Calhoun, NPS's first appointed -- and published -- scholarly author.

Dudley Knox Library / Naval Postgraduate School
411 Dyer Road / 1 University Circle
Monterey, California USA 93943

http://www.nps.edu/library

DUDLEY KNOX LIBRARY
NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA 93943

NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

APPLICATION OF LIFE DISTRIBUTIONS
TO
ESTIMATE EQUIPMENT LOSSES IN COMBAT

by

Hee Sun. Song

December 1985

Thesis Advisor: Glenn F. Lindsay

Approved for public release; distribution is unlimited.

T227284

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION
(UNCLASSIFIED)

1b. RESTRICTIVE MARKINGS

2a. SECURITY CLASSIFICATION AUTHORITY

2b. DECLASSIFICATION / DOWNGRADING SCHEDULE

3 DISTRIBUTION / AVAILABILITY OF REPORT
Approved for public release; distribution
unlimited.

4. PERFORMING ORGANIZATION REPORT NUMBER(S)

5. MONITORING ORGANIZATION REPORT NUMBER(S)

6a. NAME OF PERFORMING ORGANIZATION
Naval Postgraduate School

6b. OFFICE SYMBOL *(If applicable)*
Code 55

7a. NAME OF MONITORING ORGANIZATION
Naval Postgraduate School

6c. ADDRESS *(City, State, and ZIP Code)*
Monterey, California 93943-5100

7b. ADDRESS *(City, State, and ZIP Code)*
Monterey, California 93943-5100

8a. NAME OF FUNDING / SPONSORING ORGANIZATION

8b. OFFICE SYMBOL *(If applicable)*

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER

8c. ADDRESS *(City, State, and ZIP Code)*
Naval Postgraduate School
Monterey, California 93943-5100

10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION N

11 TITLE *(Include Security Classification)*
APPLICATION OF LIFE DISTRIBUTIONS TO ESTIMATE EQUIPMENT LOSSES IN COMBAT

12. PERSONAL AUTHOR(S)
Song, Hee S.

13a. TYPE OF REPORT
Master's Thesis

13b TIME COVERED
FROM ________ TO ________

14 DATE OF REPORT *(Year, Month, Day)*
1985 December

15 PAGE COUNT
50

16. SUPPLEMENTARY NOTATION

17. COSATI CODES

FIELD	GROUP	SUB-GROUP

18. SUBJECT TERMS *(Continue on reverse if necessary and identify by block number)*
Combat active replacement factor (CARF), life distribution
loss process, mean-time-to-loss (MTTL)

19 ABSTRACT *(Continue on reverse if necessary and identify by block number)*

Combat active replacement factors, or CARFs, are logistics planning factors currently use by the U.S. military as estimates of equipment losses in future conflicts. This thesis employs mean-time-to-loss (MTTL) estimates for various equipment types, and several scenar oriented models are given for mapping these values into CARFs. The CARF generation model can be applied for the exponential distribution or other types of life distributions such a Weibull or gamma, or a nonhomogeneous Poisson process. CARF values for several kinds of scenarios for combat losses were investigated using the loss processes for a specific life distribution. Computational results are provided.

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT
☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS

21. ABSTRACT SECURITY CLASSIFICATION
unclassified

22a. NAME OF RESPONSIBLE INDIVIDUAL
Glenn F. Lindsay

22b TELEPHONE *(Include Area Code)*
(408) 646-2688

22c. OFFICE SYMBOL
Code 55Ls

DD FORM 1473, 84 MAR

83 APR edition may be used until exhausted.
All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

Approved for public release; distribution is unlimited.

Application of Life Distributions
to
Estimate Equipment Losses in Combat

by

Hee Sun Song
Major, Republic of Korea Army
B.S., Korea Military Academy, 1975

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
December 1985

ABSTRACT

Combat active replacement factors, or CARFs, are logistics planning factors currently used by the U.S. military as estimates of equipment losses in future conflicts. This thesis employs mean-time-to-loss (MTTL) estimates for various equipment types, and several scenario-oriented models are given for mapping these values into CARFs. The CARF generation model can be applied for the exponential distribution or other types of life distributions such as Weibull or gamma, or a nonhomogeneous Poisson process. CARF values for several kinds of scenarios for combat losses were investigated using the loss processes for a specific life distribution. Computational results are provided.

TABLE OF CONTENTS

I. INTRODUCTION 8

II. LOSS PROCESS CHARACTERISTICS 10
  A. LOSS PROCESS FOR THE EXPONENTIAL 10
  B. LOSS PROCESS FOR OTHER LIFE DISTRIBUTIONS . . 11
    1. Loss Process for the Weibull 12
    2. Loss Process for the Gamma 13
  C. NONHOMOGENEOUS POISSON PROCESS OF THE LOSS . . 14

III. CARF GENERATION MODEL FOR THE EXPONENTIAL LIFE DISTRIBUTION 16
  A. CARFS WHEN ITEMS HAVE THE SAME MTTL 16
  B. CARFS WHERE NOT ALL ITEMS HAVE THE SAME MTTL 17
  C. CARFS WHERE THERE IS A CHANGE IN THE MTTL . . 17
  D. CARFS WHERE AN ON-LINE ITEM IS REPLACED BY A PREVIOUSLY INVULNERABLE ITEM 18

IV. CARF GENERATION MODEL FOR OTHER LIFE DISTRIBUTIONS 22
  A. MODEL FOR THE WEIBULL DISTRIBUTION 22
    1. CARFs When Items Have the Same MTTL . . . 22
    2. CARFs Where Not All Items Have the Same MTTL 23
    3. CARFs Where There is a Change in the MTTL 24
  B. MODEL FOR THE GAMMA DISTRIBUTION 25
    1. CARFs When Items Have the Same MTTL . . . 25
    2. CARFs Where Not All Items Have the Same MTTL 26

3. CARFs Where There is a Change in the MTTL 27

V. CARF GENERATION MODEL FOR A NONHOMOGENEOUS POISSON PROCESS 30
A. CARFS WHEN ALL ITEMS HAVE THE SAME INTENSITY FUNCTION 30
B. CARFS WHERE NOT ALL ITEMS HAVE THE SAME INTENSITY FUNCTION 33
C. CARFS WHERE THERE IS A CHANGE IN THE INTENSITY FUNCTION 33
D. CARFS WHERE AN ON-LINE ITEM IS REPLACED BY A PREVIOUSLY INVULNERABLE ITEM 35

VI. COMPARISONS AND CONCLUSIONS 39
A. COMPARISONS OF CARFS FOR EACH LIFE DISTRIBUTION 39
1. CARFs When Items Have the Same MTTL . . . 39
2. CARFs Where Not All Items Have the Same MTTL 39
3. CARFs Where There is a Change in the MTTL 40
B. CONCLUSIONS 42

APPENDIX A: CARFS WITH THE SAME MTTL (ALPHA = 2.0) . . . 44

APPENDIX B: CARFS WITH THE SAME MTTL (ALPHA = 3.0) . . . 45

APPENDIX C: CARFS WHERE NOT ALL ITEMS HAVE THE SAME MTTL 46

APPENDIX D: CARFS WHERE THERE IS A CHANGE IN THE MTTL 47

LIST OF REFERENCES 48

INITIAL DISTRIBUTION LIST 49

LIST OF TABLES

1. EXAMPLES SHOWING THE IMPACT ON CARF VALUES OF ASSUMING INFINITE SPARES FOR A HOMOGENEOUS POISSON PROCESS 20

2. EXAMPLES SHOWING THE IMPACT ON CARF VALUES OF ASSUMING INFINITE SPARES FOR A NONHOMOGENEOUS POISSON PROCESS 37

LIST OF FIGURES

3.1 The Impact of Assuming Infinite Spares on CARF Values 21
5.1 Linear Intensity Function 31
5.2 Quadratic Form of the Intensity Function 32
5.3 Linearly Increasing, then Decreasing Intensity Function 34
5.4 The Constant Loss Rate Changed During the Combat Period 35
5.5 The Impact of Assuming Infinite Spares on CARF Values 38
6.1 The Association Between CARFs and MTTLs When Items Have the Same MTTL 40
6.2 The Association Between CARFs and MTTLs When Not All Items Have the Same MTTL 41
6.3 The Association Between CARFs and MTTLs Where There is a Change in the MTTL 41

I. INTRODUCTION

Combat active replacement factors, or CARFs, are logistics planning factors currently used by the U.S Marine Corps as estimates of equipment losses in future conflicts. Their values have a significant impact on procurement, stockpiling, and plans for shipping requirements. A replacement factor is defined as " the estimated percentage of equipment in use that will require replacement during a given period due to wear-out beyond repair, enemy action, abandonment, pilferage, and other causes, except catastrophies" [Ref. 1]. Further, combat active replacement factors are applied for units during those periods when they are actually in active combat operations. A force in contact with the enemy is considered to be active combat.

There are alternatives to the use of computerized war games and simulation to obtain values for CARFs. It is possible to produce estimates directly, perhaps using professional military judgement and experience. In some cases CARF values so generated may be preferred to those obtained from combat models, since there may be more clarity about what the number was based upon, and what considerations went into its estimation [Ref. 2].

A variation on estimating CARFs directly is to use professional judgement to estimate a related measure, the mean time until loss or MTTL. The MTTL is the average time one would expect an item to survive in the combat environment.

This thesis discusses some ways of generating CARF values without to resort to war games or to simulation or direct estimation. Basic to this method is the estimation of values for MTTL, the mean time until loss, or the estimation of the intensity function for a nonhomogeneous Poisson process, and the method will be most applicable to situations where the MTTL can be estimated with more confidence than the related CARF can be directly estimated.

There are three major types of life distributions that a certain item might follow in the real world. These distributions are the exponential, the Weibull, and the gamma. First, we will look at the loss process characteristics of life distributions and then derive the CARF expressions for several kinds of scenarios according to each life distribution. These scenarios by which losses could occur in combat are:

1. Cases where all items of that type are vulnerable initially and throughout the combat period, at the same MTTL values,
2. Cases where subsets of the items in use have different MTTL values,
3. Cases where the MTTL changes at some designated time during the combat period, and
4. Cases where one item is vulnerable initially and throughout the combat period at the same MTTL, but replacements are not vulnerable until put into use.

Regardless of the life distribution, we will also discuss the case where an item's loss process follows a nonhomogeneous Poisson process at a loss rate that is time dependent.

In the following chapter, we will discuss loss process characteristics for each life distribution, and for a nonhomogeneous Poisson process.

II. LOSS PROCESS CHARACTERISTICS

Differences in the loss process may be portrayed by several kinds of life distributions. The loss rate is constant for the exponential, but for the Weibull and gamma distributions, changing loss rates can be represented by the distribution parameters. In this chapter we will look first at some loss process characteristics for each life distribution and then for a nonhomogeneous Poisson process.

A. LOSS PROCESS FOR THE EXPONENTIAL

The exponential distribution is in several senses the most fundamental distribution in reliability theory. We will look at a loss process where the event that an item is lost is independent of the loss of any other on-line item of the same kind, and where the chance of a surviving item being lost on the next day is independent of how many days it has already been in combat. This means that the individual item's loss rate may be considered constant over the period of time we are examining. For constant loss rates and independent losses, the probability distribution for the time an item survives in combat will be the exponential distribution [Ref. 2]. The expected value of this random variable is the item's MTTL. Let T be the combat survival time for a specific item, and let R be the scale parameter for the exponential. (For the exponential distribution, R is also the loss rate, with units of items per day.) The density function for the time t until the item is lost is exponential:

$$f(t) = Re^{-Rt}, \qquad R \geq 0,\ t > 0 .$$

From this we may immediately obtain the probability that the item is lost on or before time t, F(t), as

$$F(t) = p(T \le t) = 1 - e^{-Rt}, \qquad t \ge 0 .$$

We also have 1 - F(t), the probability that the item is still surviving just after time t, as

$$1 - F(t) = p(T > t) = e^{-Rt} .$$

Since the expected value or average value of an exponentially distributed random variable is the reciprocal of its parameter, we now have the mean time until loss MTTL as

$$MTTL = 1/R .$$

Accordingly, the probability that the item is not lost during D days of combat is

$$p = e^{-D/MTTL} . \tag{2.1}$$

In the following section, we will look at some loss process characteristics for the Weibull and gamma distributions.

B. LOSS PROCESS FOR OTHER LIFE DISTRIBUTIONS

Other parametric families of life distributions arising in combat situations may be constructed by assuming a loss rate as decreasing or increasing rather than being constant.

In each case, the loss rate is considered monotone over the time. For decreasing or increasing loss rates, useful probability distributions for the time an item survives in combat are the Weibull or gamma . We will look first at the Weibull distribution.

1. Loss Process for the Weibull

The loss rate r(t) for the Weibull distribution with shape parameter α and scale parameter R is

$$r(t) = \alpha R(Rt)^{\alpha-1} , \qquad \alpha > 0,\ R \geq 0,\ t > 0 ,$$

and it is increasing for $\alpha > 1$, decreasing for $0 < \alpha < 1$, and constant for $\alpha = 1$ [Ref. 3]. The density function for the time t until the item is lost is

$$f(t) = \alpha R^{\alpha} t^{\alpha-1} e^{-(Rt)^{\alpha}} , \qquad t \geq 0 .$$

From this we may obtain the probability that the item is lost on or before time t, F(t), as

$$F(t) = p(T \leq t) = 1 - e^{-(Rt)^{\alpha}} , \qquad t \geq 0 .$$

We also have 1 - F(t), the probability that the item is still surviving just after time t, as

$$1 - F(t) = e^{-(Rt)^{\alpha}} .$$

We have the mean time until loss MTTL as [Ref. 4]

$$MTTL = \frac{\Gamma(1/\alpha)}{\alpha R} .$$

The scale parameter R derived from the above formula is

$$R = \frac{\Gamma(1/\alpha)}{\alpha \, MTTL} .$$

Accordingly, if time until loss is distributed according to the Weibull distribution, the probability that the item is not lost during D days of combat is

$$p = e^{-\left(\frac{D\,\Gamma(1/\alpha)}{\alpha \, MTTL}\right)^{\alpha}} . \qquad (2.2)$$

This is equivalent to (2.1) for the exponential when α is 1, since $\Gamma(1) = 1$.

2. Loss Process for the Gamma

The loss rate r(t) for the gamma distribution with shape parameter α and scale parameter R is increasing for $\alpha > 1$, decreasing for $0 < \alpha < 1$, and constant for $\alpha = 1$ [Ref. 3]. The density function for the time t until the item is lost is

$$f(t) = \frac{R^{\alpha} t^{\alpha-1} e^{-Rt}}{\Gamma(\alpha)} , \qquad \alpha > 0,\ R \geq 0,\ t \geq 0 .$$

The probability that the item is lost on or before time t, F(t), is

$$F(t) = p(T \leq t) = \int_0^t \frac{R^{\alpha} t^{\alpha-1} e^{-Rt}}{\Gamma(\alpha)} dt , \qquad t \geq 0 .$$

When α is a positive integer, F(t) may be written in closed form as [Ref. 3]

$$1 - \sum_{i=0}^{\alpha-1} \frac{(Rt)^i}{i!} e^{-Rt} , \qquad t \geq 0 .$$

The survival probability that the item is still surviving just after t, is

$$1 - F(t) = \sum_{i=0}^{\alpha-1} \frac{(Rt)^i}{i!} e^{-Rt} .$$

We have the mean time until loss MTTL as [Ref. 4]

$$MTTL = \frac{\alpha}{R} ,$$

and from the above formula, the scale parameter, R, can be derived as

$$R = \frac{\alpha}{MTTL} \quad .$$

According to this, the probability that the item is not lost during D days of combat is

$$p = \sum_{i=0}^{\alpha-1} \frac{(\alpha D/MTTL)^i}{i!} e^{-(\alpha/MTTL)D} \quad . \qquad (2.3)$$

When α is 1, this is equivalent to (2.1) for the exponential.

C. NONHOMOGENEOUS POISSON PROCESS OF THE LOSS

In this section we consider the nonhomogeneous, also called the nonstationary, Poisson process, which is obtained by allowing the loss rate to be a function of t. As a prelude to giving a definition of a nonhomogeneous Poisson process, we shall define the concept of a function f(.) being O(h). The function f(.) is said to be O(h) [Ref. 5] if

$$\lim_{h \to 0} \frac{f(h)}{h} = 0 \ .$$

In order for the function f(.) to be O(h) it is necessary that f(h)/h go to zero as h goes to zero. But if h goes to zero, the only way for f(h)/h to go to zero is for f(h) to go to zero faster than h does. That is, for h small, f(h) must be small compared to h.

According to the definition of Ross [Ref. 5], the counting process $\{N(t), t \geq 0\}$ is said to be a nonhomogeneous Poisson process with intensity function $\lambda(t)$, $t \geq 0$, if

1. $N(0) = 0$.
2. $\{N(t), t \geq 0\}$ has independent increments.
3. $p\{N(t+h)-N(t) \geq 2\} = O(h)$.

4. $p\{N(t+h)-N(t)=1\} = \lambda(t)h + O(h)$.

If we let

$$m(t) = \int_0^t \lambda(s)ds \quad ,$$

then it can be shown that

$$p\{N(t+s)-N(t)=x\}=e^{-(m(t+s)-m(t))} \frac{(m(t+s)-m(t))^x}{x!}, \quad x=0,1,... \tag{2.4}$$

In other words, $N(t+s)-N(t)$ is Poisson distributed with mean $m(t+s)-m(t)$. Thus, $N(t)$ is Poisson distributed with $m(t)$, and for this reason $m(t)$ is called the mean value function of the process. Note that if $x = 0$, there are no losses from t to $t+s$, and thus $p\{N(t+s)-N(t)=0\}$ is the probability that one item survives for this period, and we have

$$p(\text{item survives from } t \text{ to } t+s, \text{ given survival to } t)$$

$$= e^{-(m(t+s)-m(t))} \quad . \tag{2.5}$$

With these characterizations of the above general loss processes established, we will look at various loss scenarios to derive CARFs for the exponential in the following chapter.

III. CARF GENERATION MODEL FOR THE EXPONENTIAL LIFE DISTRIBUTION

The case where the life distribution for an item is exponential has been examined by Lindsay [Ref. 2], with combat active replacement factor expressions developed for several scenarios.

In this chapter we will summarize his results.

A. CARFS WHEN ITEMS HAVE THE SAME MTTL

We look first at the case where all items are committed and initially vulnerable at the same MTTL throughout the D-day period. If n items are the initial in-use amount and are committed with independent losses, then the number of items x that would be lost during the D-day period will be binomially distributed, and the average number (out of n) lost in D days is simply the mean of the binomial distribution, or

$$\text{Average number lost} = np = n(1 - e^{-D/MTTL}) \quad .$$

From this we may readily obtain a CARF value:

$$\text{CARF} = \frac{(\text{ave number lost in D days})(100)}{(\text{in-use amount})} \quad ,$$

or

$$\text{CARF} = (1 - e^{-D/MTTL})(100) \quad . \tag{3.1}$$

Another use of (3.1) would ask if the existing CARF value would yield an MTTL that seemed reasonable, using

$$\text{MTTL} = -D/\ln(1 - \text{CARF}/100) \quad .$$

The association between CARF and MTTL for this case is illustrated by the values in the exponential column of Appendix A.

B. CARFS WHERE NOT ALL ITEMS HAVE THE SAME MTTL

If all items are initially vulnerable, but with different MTTL values for the D-day period, then CARF generation from the MTTL values is an extension of the case where all items have the same MTTL.

Let proportions p1,p2,...,pk of the in-use amount, n, have mean times until loss MTTL1, MTTL2, ... , MTTLk. The average number lost in D days for any subgroup i is

$$p_i n(1 - e^{-D/MTTLi}) \quad ,$$

and thus

$$CARF = \left(\sum_{i=1}^{k} p_i (1 - e^{-D/MTTLi}) \right)(100) \quad . \tag{3.2}$$

This is, of course, simply a weighted average of CARF values from the previous case. An example of the CARF values for this case is shown in the exponential column of Appendix C.

C. CARFS WHERE THERE IS A CHANGE IN THE MTTL

It may sometimes be of interest to construct a CARF for a situation where there is a change in the combat scenario during the D-day period. One example of this is the case where the first portion of the D days is an amphibious operation, and the MTTL might subsequently change.

For this type of situation, we let "1/MTTL1" be the loss rate for the first D1 days where x1 items are lost, and "1/MTTL2" be the loss rate for the remaining (D-D1) days where x2 items are lost. Here, total losses for the D-day period are x1+x2.

Let q1 be the probability that an item would be lost during the first period D1 and q2 be the probability that an item would be lost during the remaining D-D1 days. Then

$$q1 = 1 - e^{-D1/MTTL1} ,$$

and

$$q2 = 1 - e^{-(D-D1)/MTTL2} .$$

Both x1 and x2 are binomially distributed, x1 with parameters (n,q1), and x2 with parameters for the conditional binomial distribution ((n-x1),q2). Then

$$\begin{aligned} E[x1+x2] &= E[x1] + \sum_{x1=0}^{n} (n-x1)\ q2\ p(x1) \quad . \\ &= n\ q1 + q2\ (n - n\ q1) \\ &= n[1 - (1 -q1)(1 -q2)] \quad , \end{aligned} \qquad (3.3)$$

and the CARF would be

$$CARF = (1 - e^{-(D1/MTTL1 + (D -D1)/MTTL2)})(100) \quad . \qquad (3.4)$$

The association between CARF and MTTL for this case is illustrated by the values in the exponential column of Appendix D.

D. CARFS WHERE AN ON-LINE ITEM IS REPLACED BY A PREVIOUSLY INVULNERABLE ITEM

The expressions for generating CARFs in the previous sections have all been based upon a situation where all items were initially vulnerable. A different loss process would place one item on line, and structure its (possibly repeated) replacement with heretofore invulnerable items.

If the supply of reserve items is very large, or large enough so that the chance of its exhaustion is negligible, we can structure this case by noting that the loss process is simply a sequence of exponentially distributed time intervals over a D-day period. If we ignore boundary conditions, the number of such intervals would be a Poisson distributed random variable with a mean of D/MTTL, and the CARF [Ref. 2] would be

$$\text{CARF} = \left(\frac{D/\text{MTTL}}{\text{in-use amount}} \right)(100) \quad . \tag{3.5}$$

Equation (3.5) may be a reasonable approximation for an item with a low CARF value. If the chance of running out of replacements is not negligible, however, then (3.5) will yield an overstated CARF value. This can be converted (at the cost of simplicity) as follows. Let n be the initial amount of an item, and x be the losses in D days. Then, with the Poisson probability distribution we can write

$$P_X(x) = \begin{cases} \dfrac{(D/\text{MTTL})^x \, e^{-D/\text{MTTL}}}{x!} , & x = 0,1,\ldots,n-1 \\[2ex] \displaystyle\sum_{x=n}^{\infty} \frac{(D/\text{MTTL})^x \, e^{-D/\text{MTTL}}}{x!} , & x = n \quad , \end{cases} \tag{3.6}$$

as the loss distribution. Taking expected values, this yields a CARF expression

$$\text{CARF} = \left(\frac{100}{n}\right)\left(\sum_{x=0}^{n-1} \frac{x(D/\text{MTTL})^x \, e^{-D/\text{MTTL}}}{x!} + n \left(1 - \sum_{x=0}^{n-1} \frac{(D/\text{MTTL})^x \, e^{-D/\text{MTTL}}}{x!} \right) \right) \quad . \tag{3.7}$$

Examples of CARF values for each case are shown in Table 1. Curves associated with Table 1 are displayed in Figure 3.1. It can be seen that the approximate and correct CARF values are the same when the initial amount of an item, N, is 10.

TABLE 1

EXAMPLES SHOWING THE IMPACT ON CARF VALUES OF ASSUMING INFINITE SPARES FOR A HOMOGENEOUS POISSON PROCESS

		CARF values(%)	
n	MTTL(days)	approx	correct
3	5	200.00	97.27
	10	100.00	77.60
	15	66.67	59.40
	20	50.00	47.01
	25	40.00	38.56
5	5	120.00	89.64
	10	60.00	57.31
	15	40.00	39.55
	20	30.00	29.89
	25	24.00	23.96
10	5	60.00	60.00
	10	30.00	30.00
	15	20.00	20.00
	20	15.00	15.00
	25	12.00	12.00

In the next chapter, we will look at the CARF generation model for the Weibull and gamma life distributions.

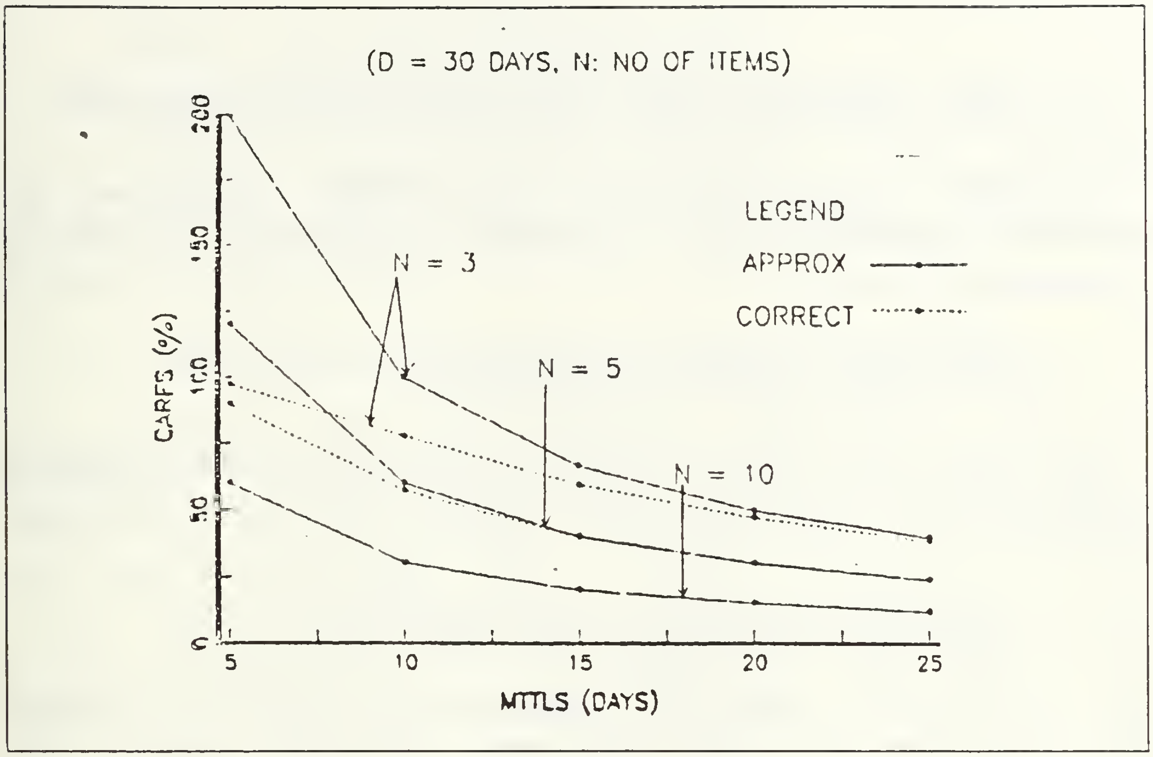

Figure 3.1 The Impact of Assuming Infinite Spares on CARF Values.

IV. CARF GENERATION MODEL FOR OTHER LIFE DISTRIBUTIONS

In this chapter, we will derive CARF expressions for several scenarios using the Weibull distribution and the gamma distribution.

A. MODEL FOR THE WEIBULL DISTRIBUTION

We are interested in looking at the Weibull distribution because it is one of the most commonly used life distributions in reliability theory.

1. CARFs When Items Have the Same MTTL

When all items are committed and initially vulnerable with the same MTTL throughout the D-day period, the probability that an item is lost during the D days is, from (2.2),

$$1 - e^{-\left(\frac{D\,\Gamma(1/\alpha)}{\alpha\ \mathrm{MTTL}}\right)^{\alpha}} .$$

Therefore,

$$\mathrm{CARF} = \left(1 - e^{-\left(\frac{D\,\Gamma(1/\alpha)}{\alpha\ \mathrm{MTTL}}\right)^{\alpha}}\right)(100) . \tag{4.1}$$

This is equivalent to (3.1) for the exponential life distribution when α is 1. Equation (4.1) permits a CARF to be computed from an MTTL estimate for the case where each item in the in-use amount is committed initially and at the same MTTL, and where losses are independent. Equation (4.1) can also be used to determine if the existing CARF value would yield an MTTL that seemed reasonable. Here we could use

$$\text{MTTL} = \frac{D\,\Gamma(1/\alpha)}{\alpha[-\ln(1-\ \text{CARF}/100)]^{1/\alpha}} \quad .$$

The association between CARF and MTTL for this case is illustrated numerically by the values in the Weibull columns of the table in Appendix A and Appendix B when α is 2 and 3, respectively.

2. CARFs Where Not All Items Have the Same MTTL

If all items are initially vulnerable, but with different MTTL values for the D-day period, then CARF generation from the MTTL values is an extension of the case where all items have the same MTTL.

For a general formula, let proportions p1,p2,...,pk of the in-use amount, n, have mean times until loss MTTL1, MTTL2, ... , MTTLk. The average number lost in D days for any subgroup i is

$$p_i n\left(1 - e^{-\left(\frac{D\,\Gamma(1/\alpha)}{\alpha\,\text{MTTLi}}\right)^{\alpha}}\right) \quad ,$$

and we have

$$\text{CARF} = \left(\sum_{i=1}^{k} p_i\left(1 - e^{-\left(\frac{D\,\Gamma(1/\alpha)}{\alpha\,\text{MTTLi}}\right)^{\alpha}}\right)\right)(100) \quad . \qquad (4.2)$$

As an example of the use of (4.2), suppose 20 % of the in-use amount have an MTTL of 100 days, half have an MTTL of 80 days, and the remaining 30 % have an MTTL of 120 days. Then from (4.2),

$$\text{CARF} = \Bigg(0.2\left(1-\ e^{-\left(\frac{30\ \Gamma(1/2)}{2\cdot 100}\right)^{2}}\right) + 0.5\left(1- e^{-\left(\frac{30\ \Gamma(1/2)}{2\cdot 80}\right)^{2}}\right)$$

$$+\ 0.3\left(1 - e^{-\left(\frac{30\ \Gamma(1/2)}{2\cdot 120}\right)^{2}}\right)\Bigg)\ (100) = 8.03\ \% \ ,$$

when α is 2 and the combat period D is 30 days. This is, of course, simply a weighted average of CARF values from the previous case. An example of the CARF values for this case is illustrated by the values in the Weibull column of Appendix C.

3. CARFs Where There is a Change in the MTTL

We next look at the scenario where the Weibull distribution for time until loss changes at time D1, which we shall represent as a change in the mean for the Weibull distribution from MTTL1 to MTTL2. The structure we will follow will assume that the loss rate after the mean changes to MTTL2 begins with argument D1, rather than 0. The survival function of a life length T for the Weibull distribution with shape parameter α and scale parameter R is

$$\bar{F}(t) = p(T>t) = e^{-(Rt)^{\alpha}}, \quad \alpha>0,\ R>0,\ t\geq 0 .$$

Let $\bar{F}_i(t)$ be the probability that an item with loss rate $r_i(t)$ survives to time t for i = 1,2. From (2.2), the survival probability $\bar{F}_1(D1)$ that an item survives to time D1 is

$$e^{-\left(\frac{D1\ \Gamma(1/\alpha)}{\alpha\ MTTL1}\right)^{\alpha}},$$

since $R = \Gamma(1/\alpha)/(\alpha MTTL)$. The conditional survival probability for the time period D1 to D, given survival to D1, is

$$\bar{F}_2(D) / \bar{F}_2(D1)$$

$$= e^{-\left(\frac{D\ \Gamma(1/\alpha)}{\alpha\ MTTL2}\right)^{\alpha}} \Big/ e^{-\left(\frac{D1\ \Gamma(1/\alpha)}{\alpha\ MTTL2}\right)^{\alpha}} .$$

Then the probability $\bar{F}(D)$ that an item survives to D is

$$\bar{F}(D) = \bar{F}_1(D1)\,\bar{F}_2(D) \Big/ \bar{F}_2(D1)$$

$$= e^{-\left(\frac{D1\,\Gamma(1/\alpha)}{\alpha\,MTTL1}\right)^{\alpha}} \quad e^{-\left(\frac{D\,\Gamma(1/\alpha)}{\alpha\,MTTL2}\right)^{\alpha}} \quad e^{-\left(\frac{D1\,\Gamma(1/\alpha)}{\alpha\,MTTL2}\right)^{\alpha}}$$

$$= e^{-\left[\left(\frac{D1\,\Gamma(1/\alpha)}{\alpha\,MTTL1}\right)^{\alpha} + (D^{\alpha} - D1^{\alpha})\left(\frac{\Gamma(1/\alpha)}{\alpha\,MTTL2}\right)^{\alpha}\right]} .$$

Therefore,

$$CARF = \left(1-e^{-\left[\left(\frac{D1\,\Gamma(1/\alpha)}{\alpha MTTL1}\right)^{\alpha} + (D^{\alpha} - D1^{\alpha})\left(\frac{\Gamma(1/\alpha)}{\alpha MTTL2}\right)^{\alpha}\right]}\right)(100). \tag{4.3}$$

When MTTL1 = MTTL2, this is equivalent to (4.1) for the scenario when items have the same MTTL, and when $\alpha = 1$, this reduces to the result (3.4) for exponentially distributed combat lives. The association between CARF and MTTL for this case is shown in the Weibull column of Appendix D.

B. MODEL FOR THE GAMMA DISTRIBUTION

The next case we shall consider is that when the life of an item in combat follows the gamma distribution. We will look at three combat scenarios.

1. CARFs When Items Have the Same MTTL

This is the case where all items are committed and initially vulnerable at the same MTTL throughout the D-day period. From (2.3), the probability that an item is lost during the D days is

$$1 - \sum_{i=0}^{\alpha-1} \frac{(\alpha D/MTTL)^{i}}{i!}\, e^{-(\alpha/MTTL)D} ,$$

where α is a positive integer.

Therefore,

$$\text{CARF} = \left(1 - \sum_{i=0}^{\alpha-1} \frac{(\alpha D/\text{MTTL})^{i}}{i!} e^{-(\alpha/\text{MTTL})D}\right)(100) \quad . \tag{4.4}$$

As with the Weibull distribution, when $\alpha = 1$ this is equivalent to (3.1) for the exponential distribution.

The association between CARF and MTTL for this case is illustrated numerically by the values in the gamma columns of Appendix A and Appendix B when α is 2 and 3, respectively.

2. CARFs Where Not All Items Have the Same MTTL

This is the case where not all items have the same MTTL for the D-day combat period. If all items are initially vulnerable, but with different MTTL values for the D-day period, then CARF generation from the MTTL values is an extension of the case where all items have the same MTTL.

For a general formula, let proportions p1,p2,...,pk of the in-use amount, n, have mean times until loss MTTL1, MTTL2, ... , MTTLk. The average number lost in D days for any subgroup i is

$$p_i n\left(1 - \sum_{j=0}^{\alpha-1} \frac{(\alpha D/\text{MTTLi})^{j}}{j!} e^{-(\alpha/\text{MTTLi})D}\right) ,$$

and we have

$$\text{CARF} = \left(\sum_{i=1}^{k} p_i\left(1 - \sum_{j=0}^{\alpha-1} \frac{(\alpha D/\text{MTTLi})^{j}}{j!} e^{-(\alpha/\text{MTTLi})D}\right)\right)(100). \tag{4.5}$$

This is the same as (3.2) for the exponential when α is 1.

As an example of the use of (4.5), suppose 20 % of the in-use amount have an MTTL of 100 days, half have an MTTL of 80 days, and the remaining 30 % have an MTTL of 120 days. Then

$$CARF = 100\ (0.2\ (\ 1 - \sum_{j=0}^{1} \frac{((2 \cdot 30)/100)^{j}}{j!}\, e^{-(2/100)(30)})$$

$$+\ 0.5\ (1 - \sum_{j=0}^{1} \frac{((2 \cdot 30)/80)^{j}}{j!}\, e^{-(2/80)(30)})$$

$$+\ 0.3\ (1 - \sum_{j=0}^{1} \frac{((2 \cdot 30)/120)^{j}}{j!}\, e^{-(2/120)(30)}))$$

$$= 13.81\ \%\ ,$$

when α is 2 and the combat period D is 30 days. This is simply a weighted average of the CARF values from the previous case.

Other examples of CARF values for this case and these proportions are shown in the gamma column of Appendix C.

3. CARFs Where There is a Change in the MTTL

We will look at the scenario where the gamma distribution changes at time D1, which we shall represent as a change in the mean for the gamma distribution from MTTL1 to MTTL2. As before, we will assume that the loss rate for the second distribution begins with argument D1. The survival function of a life length T for the gamma distribution with shape parameter α and scale parameter R is

$$\bar{F}(t) = p(T>t) = \sum_{i=0}^{\alpha-1} \frac{(Rt)^{i}}{i!}\, e^{-Rt}\ ,$$

where α is a positive integer. Let $\bar{F}_i(t)$ be the probability that an item with loss rate $r_i(t)$ survives to time t for i = 1,2. From (2.3), the survival probability $\bar{F}_1(D1)$ that an item survives to D1 can be obtained as

$$\sum_{i=0}^{\alpha-1} \frac{(\alpha D1/MTTL1)^i}{i!} e^{-(\alpha/MTTL1)D1} ,$$

since $R = \alpha/MTTL$. As with the Weibull distribution, the probability that an item survives for the time period D1 to D, given survival to D1, is

$$\bar{F}_2(D) \Big/ \bar{F}_2(D1)$$

$$= \left(\sum_{i=0}^{\alpha-1} \frac{(\alpha D/MTTL2)^i}{i!} e^{-(\alpha/MTTL2)D} \right)$$

$$\Big/ \left(\sum_{i=0}^{\alpha-1} \frac{(\alpha D1/MTTL2)^i}{i!} e^{-(\alpha/MTTL2)D1} \right) ,$$

and the probability that an item survives to D is

$$\bar{F}(D) = \bar{F}_1(D1)\ \bar{F}_2(D) \Big/ \bar{F}_2(D1) .$$

Then the CARF could be obtained from

$$CARF = \Bigg(1 - \left(\sum_{i=0}^{\alpha-1} \frac{(\alpha D1/MTTL1)^i}{i!} e^{-(\alpha/MTTL1)D1} \right)$$

$$\times \left(\sum_{i=0}^{\alpha-1} \frac{(\alpha D/MTTL2)^i}{i!} e^{-(\alpha/MTTL2)D} \right)$$

$$\Big/ \left(\sum_{i=0}^{\alpha-1} \frac{(\alpha D1/MTTL2)^i}{i!} e^{-(\alpha/MTTL2)D1} \right)\Bigg) (100) . \qquad (4.6)$$

When MTTL1 = MTTL2, this is the same CARF expression as (4.4) for the scenario when items have the same MTTL, and when $\alpha = 1$, this reduces to the result (3.4) for exponentially distributed combat lives. The association between

CARF and MTTL for this case is illustrated by the values in the gamma column of Appendix D.

In the next chapter, we will look at the CARF generation model for a nonhomogeneous Poisson process of an item loss.

V. CARF GENERATION MODEL FOR A NONHOMOGENEOUS POISSON PROCESS

Regardless of the life distribution, another case of interest is where an item's loss process follows a nonhomogeneous (nonstationary) Poisson process. The intensity function, $\lambda(t)$, related to this nonhomogeneous Poisson process is the loss rate as a function of t [Ref. 3]. We will look at four combat scenarios.

A. CARFS WHEN ALL ITEMS HAVE THE SAME INTENSITY FUNCTION

We will look first at the combat scenario where all items are committed and initially vulnerable at the same intensity function of the nonhomogeneous Poisson process throughout the D-day combat period. If n items are the initial in-use amount, and each has a probability q that an item would be lost during a D-day period, then the number of items that would be lost will be binomially distributed, and the average number lost in D days is the mean of the binomial distribution.

From equation (2.5), the probability q that an item would be lost before D days is

$$q = 1 - e^{-m(D)} \quad . \tag{5.1}$$

Thus, the average number of items being lost during the combat period D is the binomial mean

$$n(1 - e^{-m(D)}) \quad .$$

Therefore,

$$\text{CARF} = (1 - e^{-m(D)})(100) \quad . \tag{5.2}$$

Two cases associated with this process will be shown here according to the shapes of the intensity function. We will first look at the linear intensity function,

$$\lambda(t) = a + bt \ , \qquad a \geq 0,\ b \geq 0,\ 0 \leq t \leq D \ ,$$

where "a" is the intercept and "b" is the slope. When b = 0, this scenario could be treated as the same scenario for the exponential since the loss rate becomes constant.

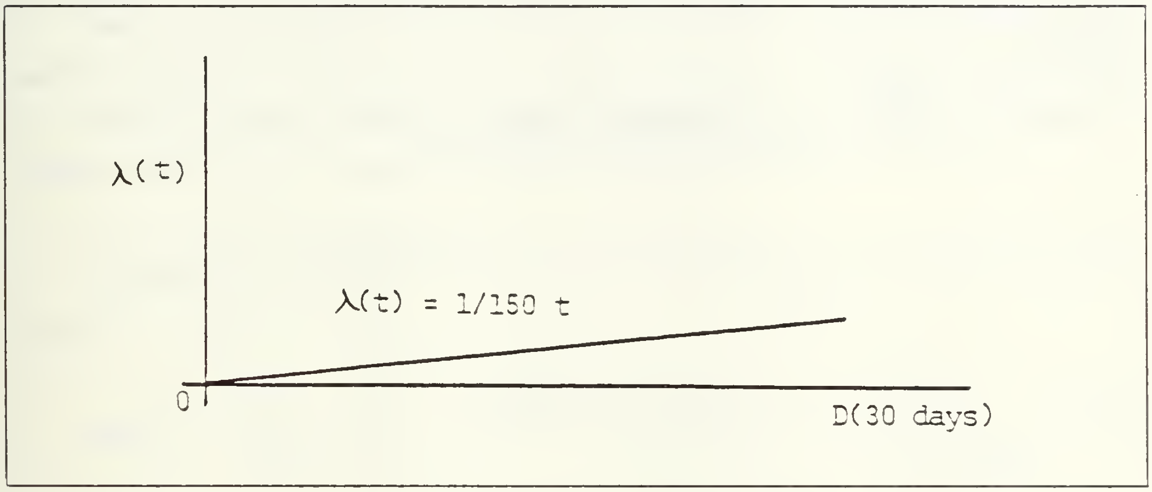

Figure 5.1 Linear Intensity Function.

For example, as shown in Figure 5.1, if a = 0, b = 1/150 and D = 30 days, then

$$\lambda(t) = (1/150)t \ ,$$

and

$$m(30) = \int_0^{30} (1/150)t \, dt = 3 \quad .$$

From this,

$$CARF = (1 - e^{-3})(100) = 95.0 \ \% \quad .$$

This example might be applicable in a combat scenario where continuous reinforcement forces on the enemy side are expected.

A quadratic form of the intensity function is the case where the intensity function is gradually increasing in the form of a quadratic as time goes on and then decreasing as shown in Figure 5.2. This might be applicable in a general combat scenario. In this case, the intensity function, $\lambda(t)$, can be expressed as

$$\lambda(t) = -a\,(t - D/2)^2 + b\ , \qquad a>0,\ b>0,\ 0 \le t \le D\ ,$$

where $D = 2\sqrt{b/a}$. The mean value function, $m(D)$, could be obtained as

$$m(D) = \int_0^D (-a\,(t - D/2)^2 + b)\,dt\ .$$

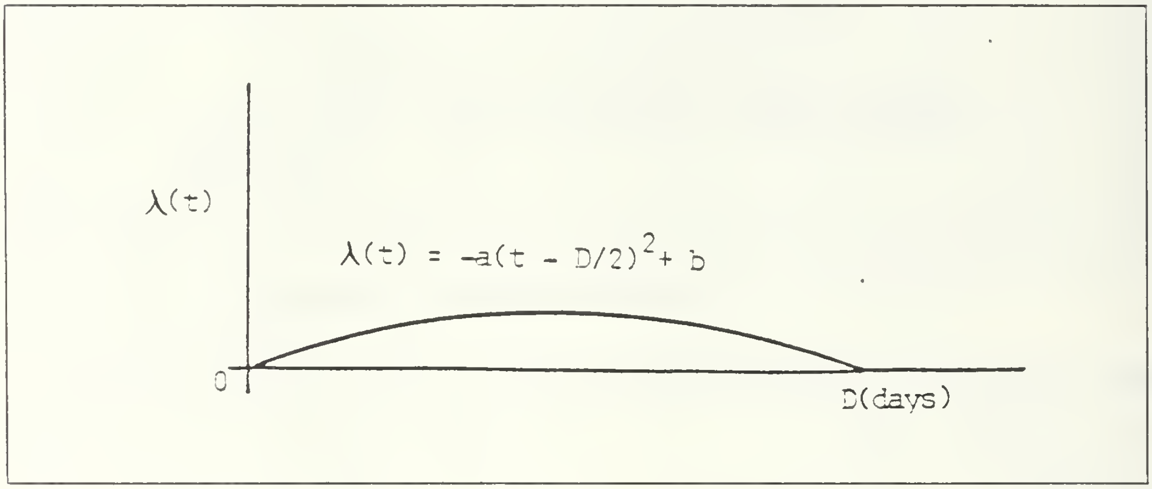

Figure 5.2 Quadratic Form of the Intensity Function.

For example, let the quadratic form of the intensity function be

$$\lambda(t) = -1/2250(t-15)^2 + 0.1\ .$$

Then

$$m(30) = \int_0^{30} (-1/2250(t-15)^2 + 0.1)\,dt = 2\ .$$

Therefore,

$$\text{CARF} = (1 - e^{-2})(100) \doteq 86.5\ \% .$$

B. CARFS WHERE NOT ALL ITEMS HAVE THE SAME INTENSITY FUNCTION

If all items are initially vulnerable, but with different intensity functions for the D-day period, then the CARF generation for this scenario is an extension of the case where all items have the same intensity function.

Let proportions $p1, p2, \ldots, pk$ of the in-use amount, n, have intensity functions $\lambda 1(t), \lambda 2(t), \ldots, \lambda k(t)$ and $mi(t)$ be a mean value function for subgroup i. Then the average number lost in D days for any subgroup i is

$$pi\ n(1 - e^{-mi(D)}) .$$

Thus,

$$\text{CARF} = \left(\sum_{i=1}^{k} pi(1 - e^{-mi(D)})\right)(100) . \tag{5.3}$$

C. CARFS WHERE THERE IS A CHANGE IN THE INTENSITY FUNCTION

The nonhomogeneous Poisson process approach to CARF development offers an opportunity to handle an intensity function which is piecewise. For example, if the loss rate is linearly first increasing and then decreasing during the D-day period, the intensity function is

$$\lambda(t) = \begin{cases} a + bt , & a \geq 0,\ b > 0,\ 0 \leq t \leq D1 \\ a' + b'(t - D1) , & a' > 0,\ b' < 0,\ D1 < t \leq D . \end{cases}$$

The mean value function can be evaluated as

$$m(D) = \int_0^{D1} (a + bt)dt + \int_{D1}^{D} (a' + b'(t - D1))dt .$$

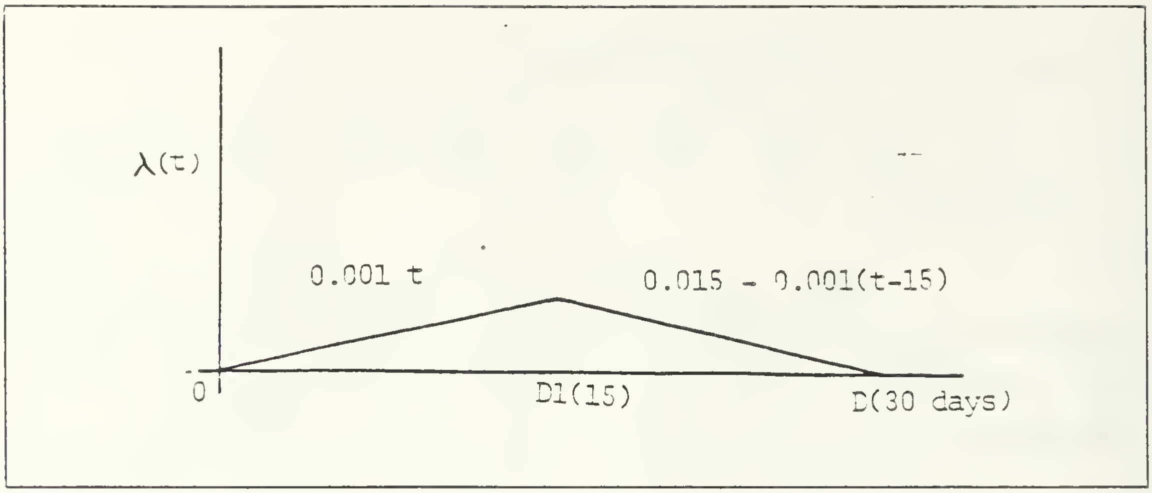

Figure 5.3 Linearly Increasing, then Decreasing Intensity Function.

For example, if the intensity function is

$$\lambda(t) = \begin{cases} 0.001t & , \quad 0 \le t \le 15 \\ 0.015 - 0.001(t-15) & , \quad 15 < t \le 30 \end{cases} ,$$

as shown in Figure 5.3, then

$$m(30) = \int_0^{15} (0.001t)dt + \int_{15}^{30} (0.015-0.001(t-15))dt = 0.225.$$

From this,

$$CARF = (1 - e^{-0.225})(100) = 20.1\ \% .$$

The case where there is a change in the constant loss rate, as shown in Figure 5.4, could also be treated as a nonhomogeneous Poisson process. However, this simply provides another way of deriving results given earlier when time to loss was exponentially distributed.

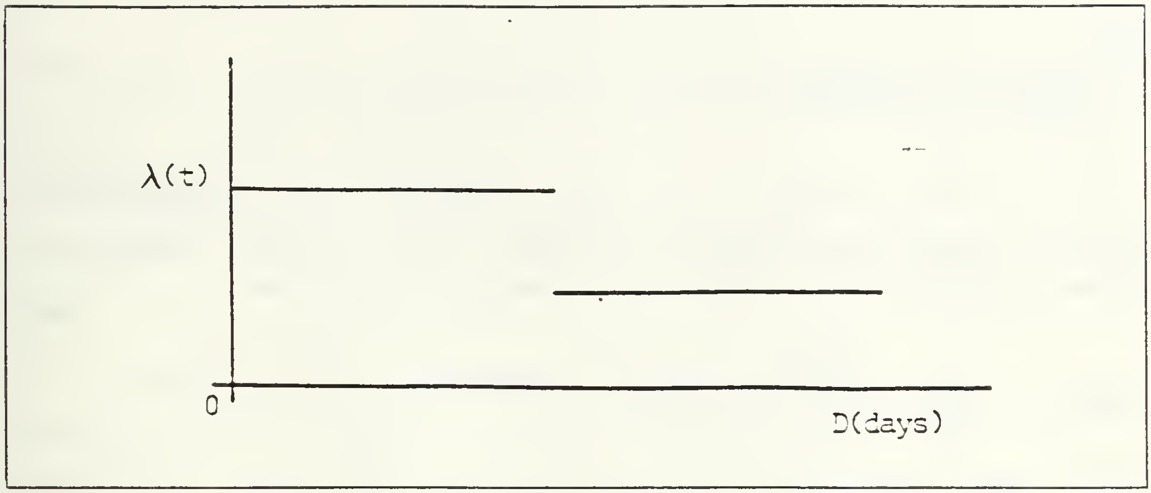

Figure 5.4 The Constant Loss Rate Changed During the Combat Period.

D. CARFS WHERE AN ON-LINE ITEM IS REPLACED BY A PREVIOUSLY INVULNERABLE ITEM

The scenario where an on-line item with an intensity function $\lambda(t)$ is replaced by heretofore invulnerable but identical items may also be examined when losses are described as a nonhomogeneous Poisson process. A loss process would place one item on line, and structure its replacement with previously invulnerable items. In other words, when an item is brought in at time t, its initial loss rate is assumed to be $\lambda(0)$ rather than $\lambda(t)$.

Let n be the initial amount of an item and x be the losses in D days. If the chance of running out of replacements is not negligible, then with formula (2.4) for the nonhomogeneous Poisson probability distribution, we can write

$$p_X(x) = \begin{cases} \dfrac{m(D)^x}{x!} e^{-m(D)} , & x = 0,1,\ldots,n-1 \\ \displaystyle\sum_{x=n}^{\infty} \dfrac{m(D)^x}{x!} e^{-m(D)} , & x = n , \end{cases} \qquad (5.5)$$

where

$$m(D) = \int_0^D \lambda(t)dt \ .$$

In a way similar to that done for the homogeneous Poisson process discussed in Chapter III, a CARF expression can be obtained by taking the expected values as

$$\text{CARF} = \left(\frac{100}{n}\right) \left(\sum_{x=0}^{n-1} \frac{x \ m(D)^x}{x!} e^{-m(D)} + n \left(1 - \sum_{x=0}^{n-1} \frac{m(D)^x}{x!} e^{-m(D)}\right)\right) . \qquad (5.6)$$

If the supply of reserve items is very large, or large enough so that the chance of its exhaustion is negligible, then the random variable X is Poisson distributed with mean m(D). Therefore,

$$\text{CARF} = \left(\frac{m(D)}{\text{in-use amount}} \right)(100) \quad , \qquad (5.7)$$

when the time period is 0 to D.

For this scenario of item replacement, it is useful to look at CARF expressions for the various forms of intensity functions introduced earlier in this chapter. As an example of a linear intensity function, let a = 0, b = 0.04, D = 30 days and n = 25. Then

$$\lambda(t) = 0.04t \ ,$$

and

$$m(30) = \int_0^{30} (0.04t)dt = 18 \quad .$$

Therefore, from (5.6)

$$\text{CARF} = \left(\frac{100}{25}\right) \left(\sum_{x=0}^{24} \frac{x \ 18^x}{x!} e^{-18} + 25\left(1 - \sum_{x=0}^{24} \frac{18^x}{x!} e^{-18}\right)\right)$$

$= 72.0\ \%$.

This value can be approximated by (5.7) which results in 71.5 %. As an example of a quadratic form of the intensity function, let n = 20 and

$$\lambda(t) = -1/225\ t^2 + 2/15\ t = -1/225\ t(t-30)\ .$$

Then

$$m(30) = \int_0^{30} (-1/225\ t^2 + 2/15\ t)dt = 20\ .$$

Therefore, the CARF = 91.12 % which is obtained from (5.6). This value could also be approximated as 100 % from (5.7).

Examples of approximate and correct CARF values are provided in Table 2.

TABLE 2

EXAMPLES SHOWING THE IMPACT ON CARF VALUES OF ASSUMING INFINITE SPARES FOR A NONHOMOGENEOUS POISSON PROCESS

		CARF values(%)	
n	m(D)	approx	correct
10	5	50.00	49.78
	10	100.00	87.49
	15	150.00	98.63
	20	200.00	99.92
15	5	33.33	33.33
	10	66.67	65.98
	15	100.00	89.76
	20	133.33	98.33
20	5	25.00	25.00
	10	50.00	49.99
	15	75.00	73.94
	20	100.00	91.12

Figure 5.5 suggests that the approximation becomes better as the ratio m(D)/n becomes small.

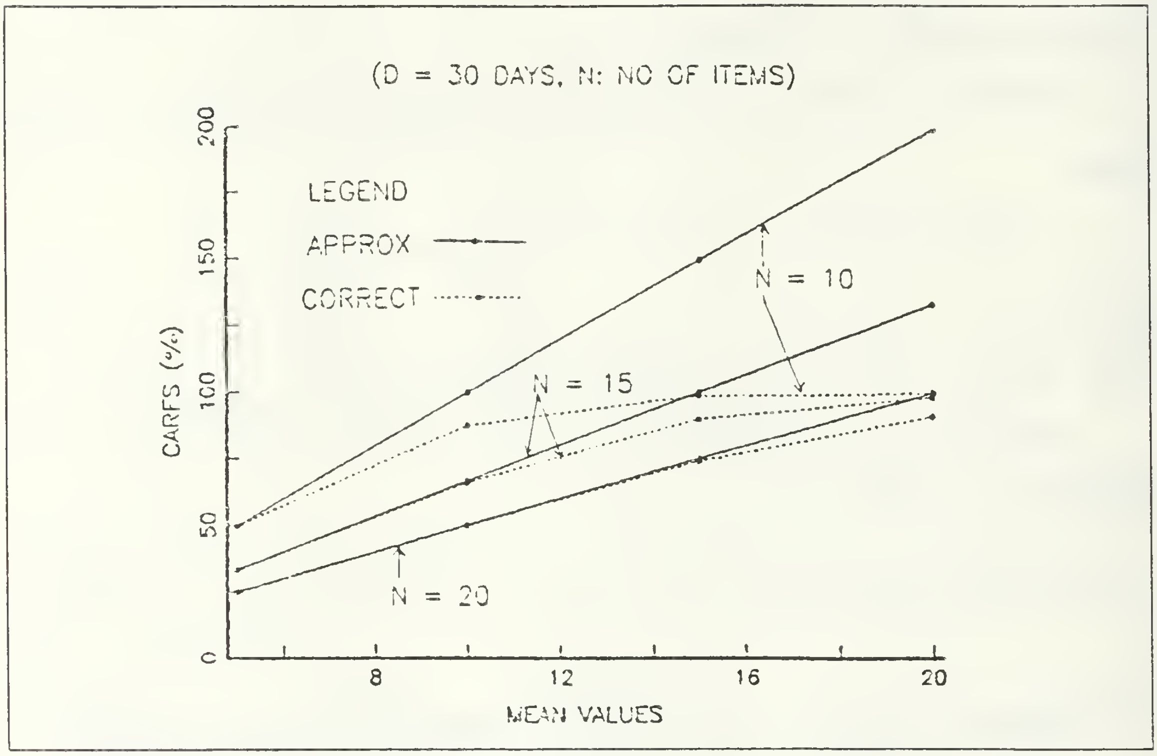

Figure 5.5 The Impact of Assuming Infinite Spares on CARF Values.

So far, we have investigated ways of finding the CARF values for various scenarios according to several types of life distributions and for a nonhomogeneous Poisson process as an item's loss process. Based on this background, we will compare CARFs for each life distribution and then suggest some conclusions in the next chapter.

VI. COMPARISONS AND CONCLUSIONS

A. COMPARISONS OF CARFS FOR EACH LIFE DISTRIBUTION

The purpose of comparisons between CARFs for each life distribution is to find the type of life distribution which can be assumed for each combat scenario when we don't know the exact life distribution of an item. Our decision rule will be that, if we don't know the exact life distribution of an item, we should use the one yielding the maximum CARF in order to avoid the risk of underestimating the CARF value. All of the following figures were made by the values provided in the appendices.

1. CARFs When Items Have the Same MTTL

The first scenario we will examine is that when all items are on line with the same MTTL.

Figure 6.1 shows CARFs as a function of MTTL values for the three distributions when all items have the same MTTL. Note that these curves are simply plots of CARFs against MTTLs and are not survival functions. The figure suggests that the CARFs for the exponential are larger than those for the Weibull and gamma (shape parameter 2.0 and 3.0) when the MTTL is approximately greater than 20. Therefore, CARF values for the exponential could be used for MTTL values greater than 25 for this combat scenario. CARFs for the Weibull distribution could be used for the MTTL values less than 25.

2. CARFs Where Not All Items Have the Same MTTL

When different subsets of the n items have different MTTL values, it is more difficult to reach general

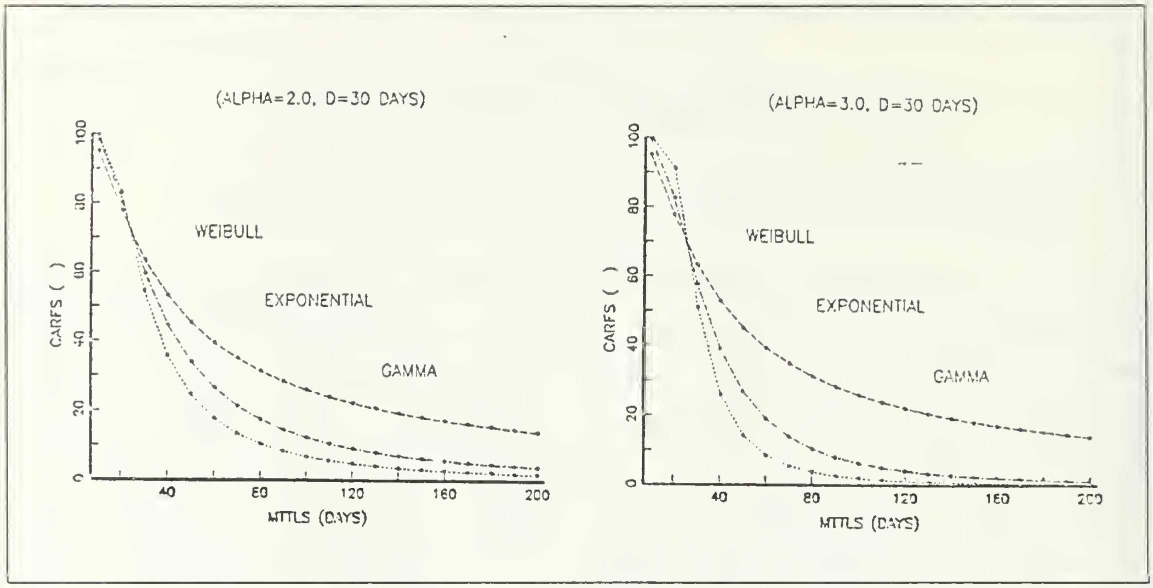

Figure 6.1 The Association Between CARFs and MTTLs When Items Have the Same MTTL.

conclusions about which distribution it might be useful to assume. Our conclusions relates to the specific example examined earlier where there were three subsets of items, each with a different MTTL. In Figure 6.2, the average of the three MTTL values was taken as an MTTL point.

From Figure 6.2 for this scenario and example, we also realize that the CARFs for the exponential are the maximum values over the range of MTTL values greater than 25. On the other hand, CARFs for the Weibull could be used for the MTTL values less than 25.

3. CARFs Where There is a Change in the MTTL

When all items have the same MTTL which changes once during the combat period, a partial comparison of distributions may be made using the numerical results developed earlier. In Figure 6.3, the average of the two MTTL values was taken as an MTTL point.

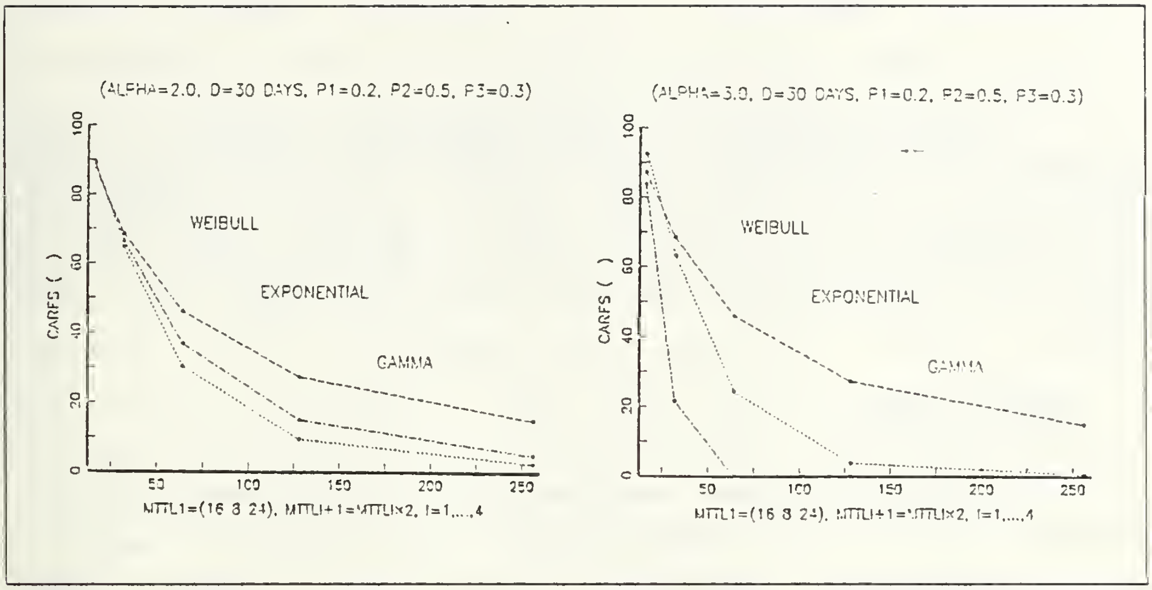

Figure 6.2 The Association Between CARFs and MTTLs When Not All Items Have the Same MTTL.

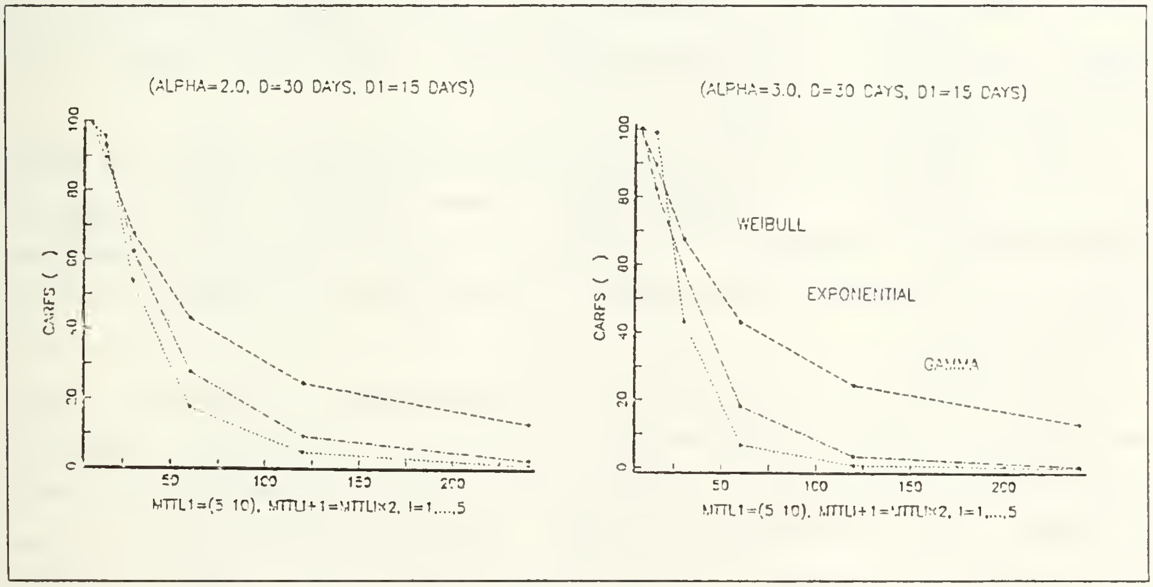

Figure 6.3 The Association Between CARFs and MTTLs Where There is a Change in the MTTL.

From Figure 6.3 for the particular example of this scenario, we could see that the CARFs for the exponential are maximum values with MTTL values greater than 20 and CARFs for the Weibull are maximum with MTTL values less than this.

B. CONCLUSIONS

We have derived some mathematical expressions for CARFs in order to estimate equipment losses in the combat environment by using the life distribution of a certain item. Furthermore, we discussed the way of finding the CARF value for the case where an item's loss follows a nonhomogeneous Poisson process.

We also attempted to suggest the kind of life distribution that would be applied for each combat scenario by comparing these values when we don't know the exact life distribution of an item.

CARF values that were drawn by this method could be used for procurement, stockpiling, and plans for shipping requirements. However, the MTTL or the intensity function for an item must be estimated or obtained in advance to use this method.

Consequently, using our conservative decision rule seeking the larger CARF value, it appears from the computations done here that the CARFs for the exponential instead of those for the Weibull or the gamma might be used if we don't know the exact life distribution of an item in which we are interested. However, for the smaller values of MTTL, CARF values for the Weibull appear to be better.

In many cases, the population of n items which are considered for CARF valuation will be spread over variable levels of combat intensity ,e.g., some items in the front and others in the rear while all are in the combat zone. It may be easier and more accurate to estimate MTTL values for each combat intensity level than to estimate an MTTL for the population as a whole. Since the proportion of the n items at each intensity level may also be well estimated in many cases, the CARF expressions for the scenario where different subsets have the different MTTL values may be most useful. The choice of scenario and life distribution will, of course, depend primarily on the specific item being studied.

The work on the preceding pages is an extension of past efforts on analytic modelling of CARF values by removing the need to assume constant loss rates or a homogeneous Poisson process. It is hoped that these extensions will be useful to those concerned with the accurate estimation of replacement factors.

APPENDIX A

CARFS WITH THE SAME MTTL (ALPHA = 2.0)

(D = 30 days)

MTTL(days)	CARF values(%)		
	Exponential	Weibull	Gamma
10.0	95.0	99.9	98.3
20.0	77.7	82.9	80.1
30.0	63.2	54.4	59.4
40.0	52.8	35.7	44.2
50.0	45.1	24.6	33.7
60.0	39.3	17.8	26.4
70.0	34.9	13.4	21.2
80.0	31.3	10.5	17.3
90.0	28.3	8.4	14.4
100.0	25.9	6.8	12.2
110.0	23.9	5.7	10.4
120.0	22.1	4.8	9.0
130.0	20.6	4.1	7.9
140.0	19.3	3.5	6.9
150.0	18.1	3.1	6.2
160.0	17.1	2.7	5.5
170.0	16.2	2.4	4.9
180.0	15.4	2.2	4.5
190.0	14.6	1.9	4.1
200.0	13.9	1.8	3.7

APPENDIX B

CARFS WITH THE SAME MTTL (ALPHA = 3.0)

(D = 30 days)

MTTL(days)	CARF values(%) Exponential	Weibull	Gamma
10.0	95.0	100.0	99.4
20.0	77.7	91.0	82.6
30.0	63.2	50.9	57.7
40.0	52.8	25.9	39.1
50.0	45.1	14.3	26.9
60.0	39.3	8.5	19.1
70.0	34.9	5.5	14.0
80.0	31.3	3.7	10.5
90.0	28.3	2.6	8.0
100.0	25.9	1.9	6.3
110.0	23.9	1.4	5.0
120.0	22.1	1.1	4.1
130.0	20.6	0.9	3.3
140.0	19.3	0.7	2.8
150.0	18.1	0.6	2.3
160.0	17.1	0.5	2.0
170.0	16.2	0.4	1.7
180.0	15.4	0.3	1.4
190.0	14.6	0.3	1.2
200.0	13.9	0.2	1.1

APPENDIX C

CARFS WHERE NOT ALL ITEMS HAVE THE SAME-MTTL

(D = 30 days, p1= 0.2, p2= 0.5, p3= 0.3)

MTTL(days)			CARF values(%)		
MTTL1	MTTL2	MTTL3	Exponential	Weibull	Gamma

(alpha = 2.0)

MTTL1	MTTL2	MTTL3	Exponential	Weibull	Gamma
16.0	8.0	24.0	87.16	89.94	88.91
32.0	16.0	48.0	68.44	64.74	66.26
64.0	32.0	96.0	45.96	30.31	36.69
128.0	64.0	192.0	27.23	9.34	14.87
256.0	128.0	384.0	14.91	2.47	4.84

(alpha = 3.0)

MTTL1	MTTL2	MTTL3	Exponential	Weibull	Gamma
16.0	8.0	24.0	87.16	92.35	83.51
32.0	16.0	48.0	68.44	63.21	21.56
64.0	32.0	96.0	45.96	24.25	0.00
128.0	64.0	192.0	27.23	3.80	0.00
256.0	128.0	384.0	14.91	0.49	0.00

APPENDIX D

CARFS WHERE THERE IS A CHANGE IN THE MTTL

(D = 30 days, D1 = 15 days)

MTTL(days)		CARF values(%)		
MTTL1	MTTL2	Exponential	Weibull	Gamma
(alpha = 2.0)				
5.0	10.0	98.89	100.00	99.85
10.0	20.0	89.46	95.46	92.89
20.0	40.0	67.53	53.84	62.36
40.0	80.0	43.02	17.57	27.69
80.0	160.0	24.52	4.72	9.28
160.0	320.0	13.12	1.20	2.68
(alpha = 3.0)				
5.0	10.0	98.89	100.00	99.81
10.0	20.0	89.46	98.90	82.57
20.0	40.0	67.53	43.07	58.53
40.0	80.0	43.02	6.80	18.24
80.0	160.0	24.52	0.88	3.59
160.0	320.0	13.12	0.11	0.56

LIST OF REFERENCES

1. U.S. Marine Corps, War Reserve Policy Manual, pp. 4.3-4.4, 1984.

2. G.F. Lindsay, An Examination of the USMC Combat Active Replacement Factor (CARF) Determination System, pp. 1-13, NPS Technical Report #NP 555-85-005, 1985.

3. R.E. Barlow and Frank Proschan, Statistical Theory of Reliability and Life Testing Probability Models, 2nd Edition, pp. 73-75, To Begin With, 1981.

4. A.M. Law and W.D. Kelton, Simulation Modelling and Analysis, pp. 159-163, McGraw-Hill, 1982.

5. S.M. Ross, Introduction to Probability Models, 2nd Edition, pp. 161-189, Academic Press, 1980.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Technical Information Center Cameron Station Alexandria, Virginia 22304-6145	2
2.	Library, Code 0142 Naval Postgraduate School Monterey, California 93943-5100	2
3.	Department Chairman, Code 55Ws Department of Operations Research Naval Postgraduate School Monterey, Ca 93943-5100	2
4.	Professer G. F. Lindsay, Code 55Ls Department of Operations Research Naval Postgraduate School Monterey, California 93943-5100	2
5.	Library P.O.Box 77 Gongneung Dong, Dobong Gu Seoul, 130-09, Korea	2
6.	Commandant of Marine Corps Code LPP HQUSMC Washington,D.C. 20380-0001	2
7.	Professor J. W. LaPatra, Code 54Lp Department of Administrative Sciences Naval Postgraduate School Monterey, California 93943-5100	1
8.	Major Hee Sun, Song kwangju Si, Wolsan 2 Dong, 254-24 17 Tong 5 Ban Seoul, 500-00, Korea	10

6002 177

215601

Thesis
S665592 Song
c.1 Application of life distribution to estimate equipment losses in combat.

215601

Thesis
S665592 Song
c.1 Application of life distribution to estimate equipment losses in combat.

thesS665592
Application of life distributions to est
3 2768 000 68793 3
DUDLEY KNOX LIBRARY